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Low Yield Curves and Absolute/Normal Volatilities

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Summary

In this paper, we look at the changes that have occurred in interest rate markets since the financial crisis. We consider how insurers can address the challenge of low and (more recently) negative yield curves as central banks have responded to challenging economic conditions with a range of unconventional monetary policies.

Different measures of swaption implied volatilities have emerged as market standards for quoting derivative prices. We review and discuss the advantages of using absolute/normal volatilities in the current interest rate environment.

Finally, the implications of a move to absolute/normal volatilities are discussed in the context of Market Consistent valuation for insurance companies.

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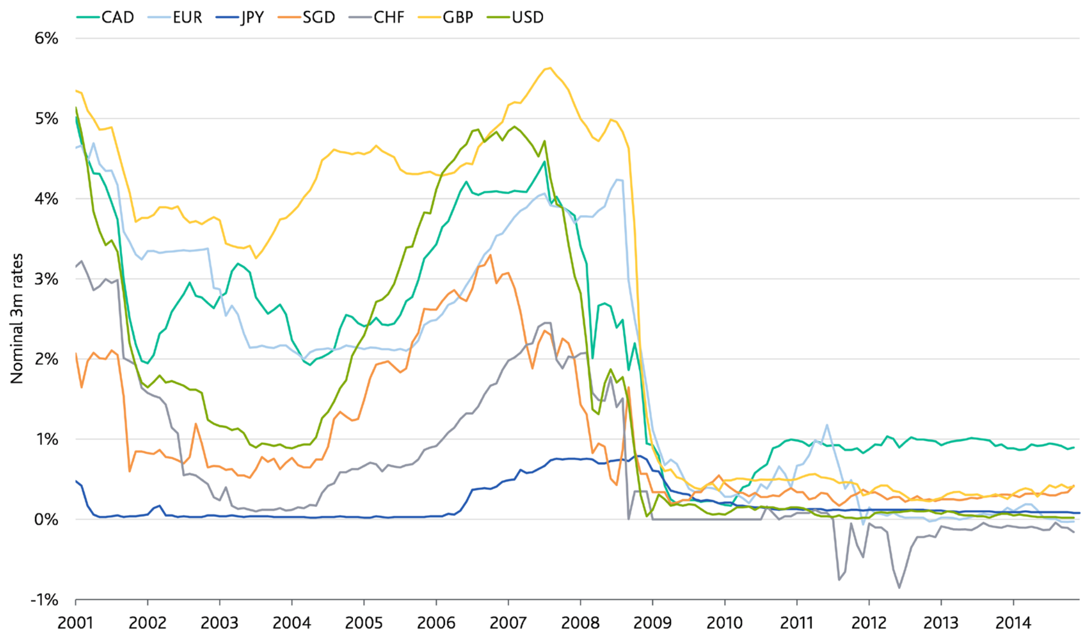
Introduction

Almost ten years after the financial crisis, interest rates, and yields curves across the globe are different from their long-term historic norm. Short rates, the mainstay of monetary policy before the crisis, have been pinned to the floor for a prolonged period. Exhibit 1 shows the accelerated fall during the crisis and the subsequent period of consistently low rates.

Exhibit 1: Short rates before and since the financial crisis.

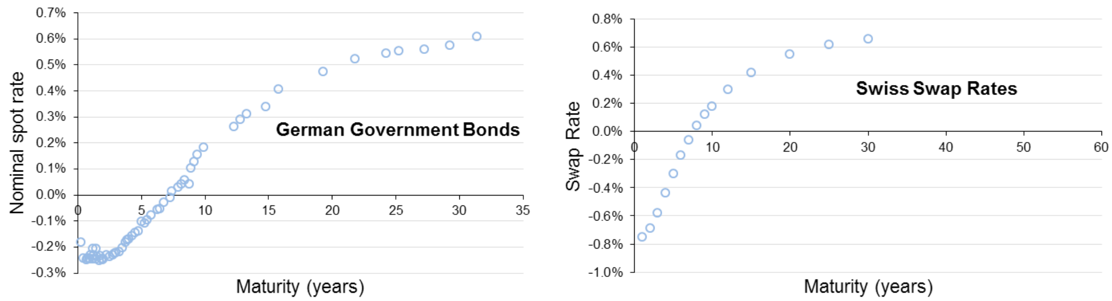


In many central banks' views, the global financial crisis led to a need for large negative real interest rates. When inflation is low, imposing negative real rates is difficult and Quantitative Easing (QE) was embraced as a workable proxy. This form of monetary policy response quickly became the norm in most of the large global economies, with QE introduced in the US and UK in 2008, and in Japan in 2011. The intention of QE was to force longer term yields lower by purchasing government bonds and other longer dated financial assets, in combination with clear future guidance that policy rates would stay low for a prolonged period of time. The impact has been an extended period of low-cost financing for households, banks, the commercial sector, and governments.

The Eurozone and its neighbors' reactions have been slower and less decisive. Between 2008 - 2014 longer term yields in the Eurozone were pulled lower by the broader global trends. However, it was not until January 2015 that the European Central Bank announced its own program of quantitative easing. The impact of this late venture into unconventional monetary loosening has been a lowering of longer term yields and rates in the Eurozone and Sweden introducing its own parallel program of easing.

Even less conventional forms of monetary response have also been tried. The most notable of these measures is the imposing of negative nominal interest rates. The fundamental barrier to setting negative real rates directly was a perceived 'hard floor' on nominal rates, with the consensus being that nominal rates could not become negative. This view is challenged by the introduction of negative nominal rates in Sweden, Switzerland, the Eurozone and Japan (exhibit 2). In these economies, it became evident that Central Banks could charge for deposits and investors would, in certain situations, pay for the security of sovereign bonds for periods of up to ten years. The U.S. Federal Reserve included a negative rate scenario in their 2016 stress tests.

Exhibit 2: Negative Yields at end Q1 2015.



One other notable Central Bank action was Switzerland's September 2011 decision to introduce, without notice, a floor on the Swiss Franc to Euro exchange rate provoking one of the largest one day moves in Foreign Exchange (FX) ever experienced. This floor was held for almost 4 years, until it was dropped days before the Eurozone QE announcements in January 2015.

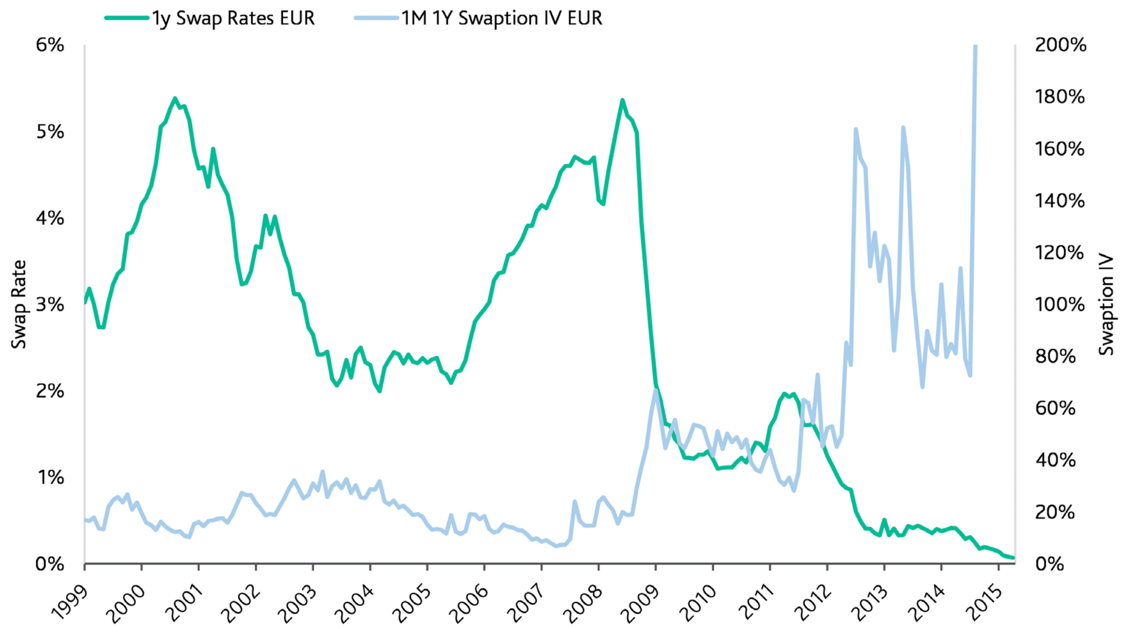
This paper aims to help understand the challenges faced in the use of Black's formula in the current economic environment of low yields curves, and demonstrate how the usage of absolute/normal implied volatilities can address them.

New Perspectives on Interest Rate Volatilities

The long standing convention of quoting derivative prices for interest rate options like caps, floors, and swaptions is to use Black's formula for option pricing which assumes a lognormal distribution for interest rates. Historical Black implied volatilities for one month into one year Euro swaptions since 1999 can be seen in Exhibit 3.

Before 2008, Black volatilities typically varied between 10% and 30% with a clear inverse relationship between the level of implied volatilities and rates. As rates fell in response to the financial crisis in 2008, Black volatilities changed. Since 2009 there appears to have been three distinctive volatility regimes. The first was between 2009 and mid-2012 when Black volatilities varied between 30% and 70%. The second (following rates cuts to about 50 basis points (bp)) was the period between mid-2012 and mid-2014 when Black volatilities varied between 80% and 170%. Finally, a further cut in rates in mid-2014 too close to zero percent led to volatilities greater than 200%. In each circumstance, the reason for the increase in volatility regime was a drop in the underlying level of rates. Similar sensitivities of Black implied volatilities to rates are seen in different markets and for different tenor/maturity combinations.

Exhibit 3: Black swaption implied volatilities for the Eurozone.



Ideally, an implied volatility measure used for pricing interest rate derivatives would hold approximately constant and vary in an intuitive way under a range of market conditions or across different ranges of instruments including:

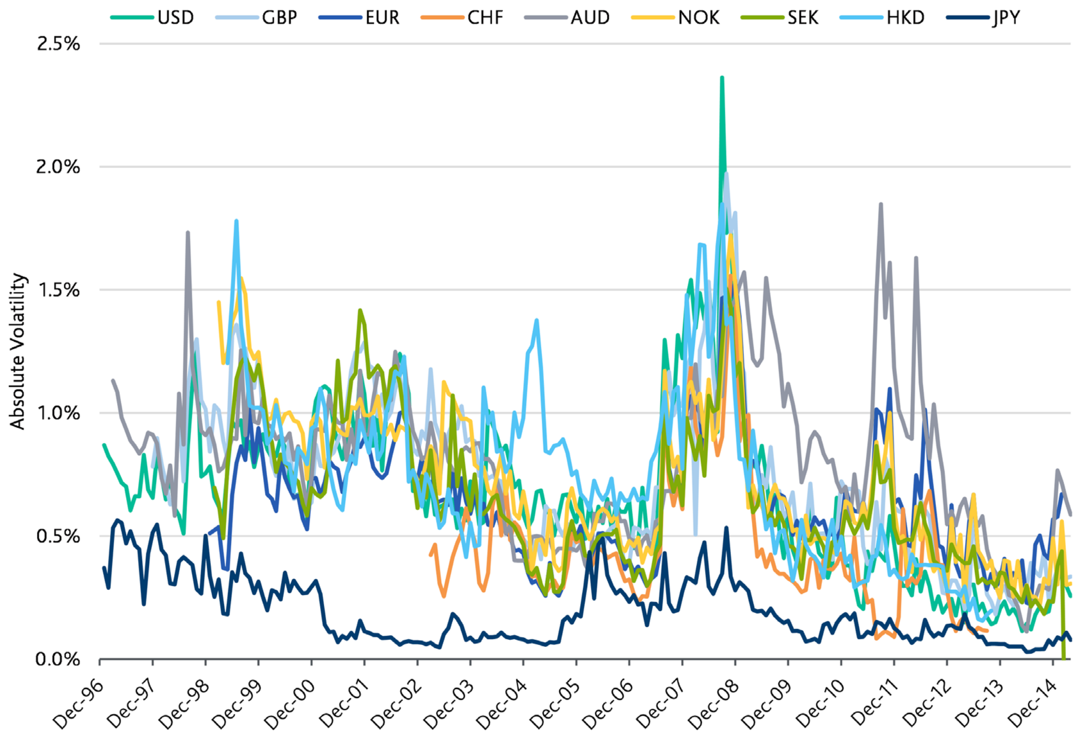
» As strike and forward rates change.

» When maturities and tenors vary.

» For different types of instrument (for example puts and calls, payers and receivers) linked via put-call parity.

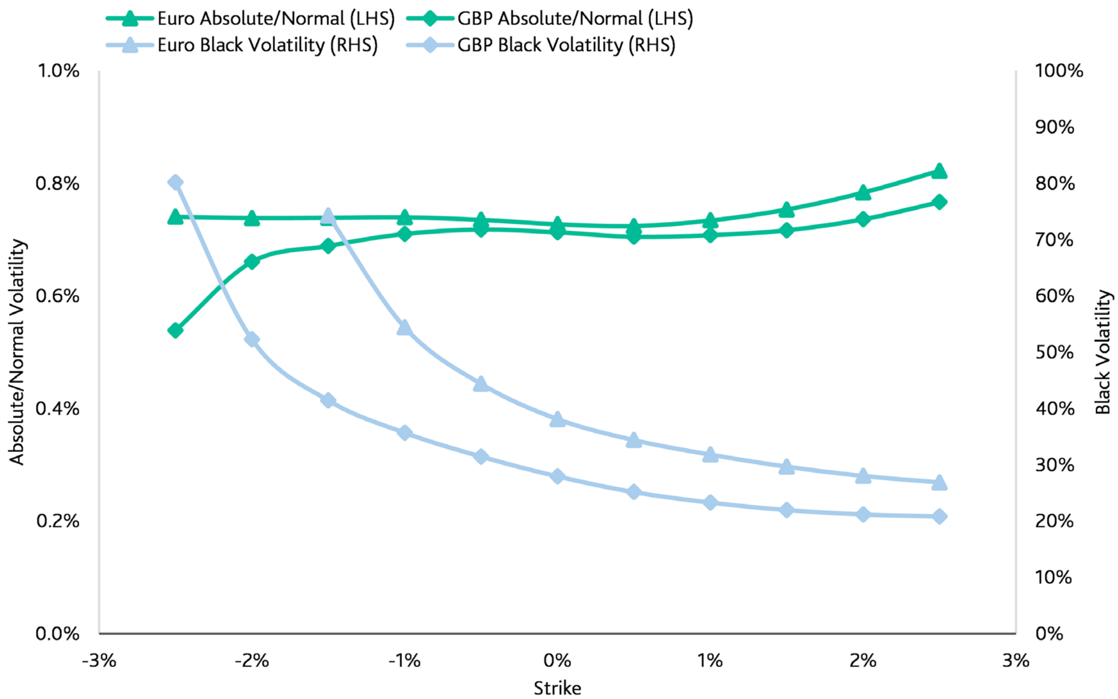
Before 2008 when rates were higher, Black implied volatilities performed reasonably well against these criteria. However, since 2009 participants in the swaption markets have increasingly chosen to use absolute/normal implied volatilities. In Exhibit 4, we see absolute volatilities for a range of different economies since 1996 (we have created the series by taking Black implied volatilities and multiplying by the prevailing swap rate). These absolute/normal implied volatilities have been considerably more stable, and only a slight positive correlation between absolute volatilities and rate levels is observed. The more stable behavior of absolute/normal rates has led to a view among many traders of interest rate derivatives that interest rate distributions are closer to normal than lognormal.

Exhibit 4: Normal/Absolute Volatilities for a range of economies since 1996.



More recently, severe limitations with Black's formula have become apparent as rates have fallen close to or (in some economies) below zero. Because Black's formula assumes that rates are lognormally distributed, the formula becomes infinitely sensitive to price changes as rates tend to zero. The formula cannot be solved at all for negative strikes or forward rates. When there is a skew of swaption volatilities for different strikes away-from-the-money, Black volatilities have shown a marked dependence on the strike level whereas by contrast the absolute/normal volatilities are relatively constant. We present an example at the end of June 2015 in Exhibit 5.

Exhibit 5: Away-from-the-money Absolute/Normal and Black Volatilities for GBP and EUR 10 into 10 swaptions at the end of June 2015



A third convention for quoting swaption implied volatilities which has also become popular is to use a 'displaced lognormal' version of Black's formula. This volatility measure has some attractive properties compared to the normal and Black formulas. There is an extra degree of freedom in specifying the displacement parameter which can be used to vary the underlying rate distribution, and it is closely related to displaced stochastic interest rate models commonly used for simulation. However, the principal reason for its adoption by some market participants appears to be convenience – the displaced lognormal pricing formula is a simple variant of Black's formula, and it can therefore be adopted with little extra effort.

The sensitivity of Black implied volatilities to levels of rates, and the subjectivity involved in choosing the displacement to quote a displaced lognormal volatility highlights the benefit of choosing absolute/normal instead of Black volatilities.

Implied Volatility	Stable?	Objective?
Black	✗	✓
Absolute/Normal	✓	✓
Displaced Lognormal	✓	✗

Implications for Insurers

If, as seems likely, the insurance industry decides to adopt absolute/normal swaption implied volatilities for their Market Consistent (MC) and Market Consistent Embedded Value (MCEV) applications there are a range of different implications for the industry.

MONEYNESS OF GUARANTEES?

Most insurance guarantees are not struck at-the-money. Many guarantees were written years ago when interest rates were higher, so many guarantees are deeply in-the-money. If an accurate away-from-the-money surface is used to value guarantees, then the choice of Black or absolute/normal volatilities is a moot-point. However, great care must be taken when valuing with models which are not capable fitting away-from-the-money. Extrapolating volatilities using a lognormal model may produce values for deep in-the-money guarantees which are biased up compared to using models better capable of fitting the away-from-the-money market surface.

WHICH SWAPTION CONVENTION TO USE WHEN YOU USE A NON-SWAP CURVE (FOR EXAMPLE EUROPEAN INSURANCE OCCUPATIONAL PENSIONS AUTHORITY (EIOPA), GOVERNMENT, AA RATED) CALIBRATIONS?

Different base curves are used for internal Economic Capital (EC) calculations, Solvency II reporting, US statutory reporting, and international accounting standards. Evaluating the impact of switching from Black volatilities to absolute/normal volatilities is relatively straightforward. When the base yield curve is lower than the swap curve (for example government curves) the value of guarantees will be higher if absolute/normal volatilities are used. If the base curve is higher than the swap curve (for example EIOPA or AA), the value of guarantees will be lower.

WHICH SWAPTION CONVENTION TO USE WITH EIOPA CURVES WITH/WITHOUT YIELD CURVE ADJUSTMENTS?

The Solvency II guidelines allow several adjustments to be made to the risk free yield curve, most of which raise the yield curve to higher levels than the standard EIOPA curve. Using Black volatilities in these calibrations produce higher valuations for guarantees than when working with absolute/normal volatilities.

WHICH SWAPTION CONVENTION IS APPROPRIATE TO USE FOR SOLVENCY II STANDARD FORMULA AND SWISS SOLVENCY TEST (SST)?

The Solvency II standard formula does not have a volatility stress. However, when the yield curve stress is applied 'volatility' is to be held constant. The SST does specify a volatility stress but the swaption volatility convention to be used is not specified. Assuming most insurers with sensitivity to volatility are performing downward yield curves stresses then the impact of holding Black Volatilities constant produces higher stressed values than absolute/normal volatilities.

WHICH SWAPTION CONVENTION TO USE WHEN CALCULATING FINANCIAL EXPOSURES LIKE EFFECTIVE DURATIONS AND CONVEXITIES?

When calculating effective durations and convexities it is normal to stress yield curves up and down, while holding volatilities constant. Using Black volatilities rather than absolute/normal volatilities lead to different values for the duration and convexity of liabilities.

EVALUATING FIT TO MARKET, VOLATILITY EXTRAPOLATION, AND VOLATILITY ANCHORING

Evaluating and validating the quality of fit to market is likely to be easier with absolute/lognormal volatilities rather than Black volatilities. This was most evident at the end of Q1 2015 in the Eurozone when Black volatilities could not be quoted for many swaption contracts due to negative strikes or forward rates and the entire surface was extremely unstable. Validating fits in terms of prices can help to evaluate the fit to market in this type of situation. However, when extrapolating surfaces either to longer maturities or away-from-the-money, working with absolute/normal volatilities can simplify the process and produce much more robust and justifiable results.

CORRELATION BETWEEN VOLATILITY AND YIELD CURVES

Correlations between yield curve levels and Black volatilities are typically strongly negative. Correlations between yield curve levels and absolute/normal volatilities are weakly positive.

RISK DECOMPOSITION

When combining yield curve and volatility stresses for capital calculation stresses, Black volatilities are large and offset a downward yield curve stress. Stresses to absolute volatilities are smaller and tend to reinforce the yield curve stress.

HEDGING INTEREST RATE RISK

Using Black volatilities may lead to more emphasis on volatility hedging than is justified, with much of the effort resulting in positions which offset yield curve hedges. Using absolute/normal volatilities makes it easier to focus effort on hedging the underlying yield curve movements.

REAL WORLD MODELING

Real world stresses based on lognormal assumptions for the distribution of rates have to be treated with caution. For example, the Solvency II Standard Formula Stresses for yield curves are arguably weak given the current yield curve environment. Internal models, where interest rate stresses are based on lognormal assumptions, may come under significant scrutiny both in terms of how they perform given current yield curve environment and their backdated performance. Many users of internal models, which have interest rate volatility stresses, may also need to revisit the calibration of this part of the model. Although time consuming, insurers who choose to review their calibration may find a move to absolute/normal volatilities significantly simplifies their modeling assumptions.

ALIGNMENT WITH THE BANKING SECTOR

The banking sector appears to have been working with a dual convention of Black and absolute/normal volatilities for many years. It is unlikely that the adoption of the newer convention happened for any reason other than it was considered superior for many applications. Anecdotal evidence is that most banks prefer absolute/volatilities for internal use, and provide Black volatilities as a price quote largely due to demand from clients using this convention.

Conclusion

The changes that have occurred in interest rate markets since the financial crisis with low and negative yields curves is a major challenge for most insurers who have historically used Black's formula for interest rate option pricing.

In this paper, we presented the issues observed when low interest rates occurred and analyzed the three distinctive volatility regimes between 2009 and mid 2014 leading to Black's volatilities greater than 200%. The main reason of this increase was a drop in the underlying level of rates due to central bank intervention with both the base rate and quantitative easing.

Also, we have outlined the importance of using implied volatilities which performed reasonably well when rates were higher (before 2008) and when rates became low (since 2009). Finally, we discussed the potential impacts of this decision in the context of Market Consistent valuation for insurance companies.

Overall, we suggest the use of absolute/normal implied volatilities in the current low interest rate environment. This conclusion is based on our research that normal/absolute volatiles are more robust across a range of interest rate regimes.

Appendix – Swaption Implied Volatility Formulas

Lognormal/Black Swaption Formula

$$\frac{\mathrm{d}F_t^T}{F_t^T} = \sigma \mathrm{d}W_t.$$

Then the price of a payer swaption (or equity call option) is

$$\begin{aligned} c_t &= P(t,T)\mathrm{E}_{\mathbb{Q}}[(F_T^T - K)_+ | \mathcal{F}_t] \\ &= P(t,T)\mathrm{E}_{\mathbb{Q}}\left[\left(F_t^T \exp\left(-\frac{1}{2}\sigma^2(T-t) + \sigma(W_T - W_t)\right) - K\right)_+ \middle| \mathcal{F}_t\right] \\ &= P(t,T)(F_t^T N[d_+(T-t, F_t^T)] - KN[d_-(T-t, F_t^T)]), \end{aligned}$$

where

$$\begin{aligned} d_\pm(\tau, F) &:= \frac{\ln\left(\frac{F}{K}\right) \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}, \\ N[d] &:= \int_{-\infty}^{d} \phi(z)\mathrm{d}z, \\ \phi(z) &:= \frac{1}{\sqrt{2\pi}} e^{-z^2/2}. \end{aligned}$$

Normal/Absolute Swaption Formula

$$\mathrm{d}F_t^T = \sigma \mathrm{d}W_t.$$

The price of a payer swaption (or equity call option) is

$$\begin{aligned} c_t &= P(t,T)\mathrm{E}_{\mathbb{Q}}[(F_T^T - K)_+ | \mathcal{F}_t] \\ &= P(t,T)\mathrm{E}_{\mathbb{Q}}[(F_t^T + \sigma(W_T - W_t) - K)_+ | \mathcal{F}_t] \\ &= P(t,T)\int_{-\infty}^{\infty} \left(F_t^T - K + \sigma\sqrt{T-t}z\right)_+ \phi(z)\mathrm{d}z \\ &= P(t,T)\sigma\sqrt{T-t}\int_{-\infty}^{z_t} (z_t - z)\phi(z)\mathrm{d}z \\ &= P(t,T)\sigma\sqrt{T-t}\left(z_t\int_{-\infty}^{z_t} \phi(z)\mathrm{d}z - \int_{-\infty}^{z_t} z\phi(z)\mathrm{d}z\right) \\ &= P(t,T)\sigma\sqrt{T-t}\big(z_t N[z_t] + \phi(z_t)\big) \\ &= P(t,T)\left((F_t^T - K)N[z_t] + \sigma\sqrt{T-t}\phi(z_t)\right), \end{aligned}$$

Where

$$z_t := (F_t^T - K)/\sigma\sqrt{T-t}.$$

Displaced Lognormal Swaption Formula

$$\frac{\mathrm{d}\tilde{F}_t^T}{\tilde{F}_t^T} = \sigma \mathrm{d}W_t,$$

where $\tilde{F}_t^T \coloneqq F_t^T + \delta$. The price of a payer swaption (or equity call option) is

$$\begin{aligned} c_t &= P(t,T)\mathrm{E}_{\mathbb{Q}}\left[(F_T^T - K)_+ | \mathcal{F}_t\right] \\ &= P(t,T)\mathrm{E}_{\mathbb{Q}}\left[\left(\tilde{F}_T^T - \tilde{K}\right)_+ \middle| \mathcal{F}_t\right] \\ &= P(t,T)\left(\tilde{F}_t^T N\left[d_+\left(T-t,\tilde{F}_t^T\right)\right] - \tilde{K} N\left[d_-\left(T-t,\tilde{F}_t^T\right)\right]\right), \end{aligned}$$

Where

$$\begin{aligned} \tilde{K} &\coloneqq K + \delta. \\ \tilde{F}_t^T &\coloneqq F_t^T + \delta. \end{aligned}$$